Extra material to *Micro-cosmos model of a nucleon*

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Abstract

This set of problems[1] refer to the paper "Micro-cosmos model of a nucleon" [1]. They can be useful while reading the paper and working on developing the model. Solutions to the problems will be provided on request. Equation numbers in bold refer to the paper.

Topics covered in the problems:

- **The cyclic micro-cosmos.** Solution of the Friedmann equation for a closed radiation-dominated model universe. Scale factor written in geometric units.
- **Mass and radius of micro cosmos.** Definition of a time-varying mass and calculation of the total mass integrated over one cycle. The de Broglie relation.
- **The proton.** Deriving a mass-radius relation for the proton.
- **Density of micro-cosmos**. Nuclear density and radial density.

References

[1] Michael Cramer Andersen (2024), Micro-cosmos model of a nucleon. *Modern Physics Letters A* vol. **39** (in press). Preprint on micro.cozmo.dk.

[2] Barbara Ryden (2003), Introduction to Cosmology. Addison-Wesley.

[1]Cite as: M.C. Andersen (2024), Extra material to Micro-cosmos model of a nucleon. micro.cozmo.dk.

1 The cyclic micro-cosmos

The paper "Micro-cosmos model of a nucleon" [arXiv ref.] [1] builds on classical cosmology with the standard Friedmann equation. Equation **(1)** in the paper is known from various cosmology textbooks e.g. [2] section 6:

$$\frac{H^2}{H_0^2} = \Omega_m \left(\frac{a_0}{a}\right)^3 + \Omega_r \left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_k \left(\frac{a_0}{a}\right)^2 \tag{1}$$

By introducing the maximal size $a_{\max}$ we eliminate Ω_0 and arrive at equation **(2)**:

$$\left(\frac{da}{dt}\right)^2 = H_0^2 \cdot \frac{a_{\max}^2 \cdot a_0^4}{a_{\max}^2 - a_0^2} \cdot \left(\frac{1}{a^2} - \frac{1}{a_{\max}^2}\right) \tag{2}$$

By introducing the cycle time t_{crunch} we eliminate H_0 and the equation is solved for the scale factor $a(t)$:

$$a(t) = \sqrt{a_0^2 + 4 \cdot (a_{\max}^2 - a_0^2) \cdot \left(\frac{t}{t_{\text{crunch}}}\right) - 4 \cdot (a_{\max}^2 - a_0^2) \cdot \left(\frac{t}{t_{\text{crunch}}}\right)^2} \tag{3}$$

Problems:

1. Consult your favorite cosmology reference source and make sure that you understand equation **(1)**.
2. Derive the expression for $a_{\max}$ by setting $H = 0$ and solving for a.
3. Use $a_{\max}$ to eliminate Ω_0 and derive equation **(2)**.
4. Derive the expression for t_{crunch}.
5. Solve the second order differential equation **(2)** by separation of the variables t and a and integrate. Hints: Each side is integrated from a minimum value to the integration variable. Use t_{crunch} to eliminate H_0 as late as possible. The following integral is useful: $\int \frac{da}{\sqrt{1/a^2 - C}} = \frac{1}{C}(1 - \sqrt{1 - C \cdot a^2})$ for $a > 0$ with $C = 1/a_{\max}^2$.
6. Show that $a(t)$ satisfies the differential equation **(2)**.

2 Mass and radius of micro-cosmos

It is not common to calculate the total mass of a model universe describing the macroscopic world. Observable properties are defined for an observer that is located inside the universe and nothing can escape the cosmological horizon. However a micro-cosmos that is used to describe a quantum particle must be embedded within a nearly flat spacetime background and we may define the mass anyway.

Multiplying radiation density and volume leads to the time-varying mass, equation **(6)**:

$$m_{\mathrm{mc}}(t) = \rho_r \cdot V = \frac{l_{\mathrm{Pl}} \cdot m_{\mathrm{Pl}}}{a(t)} \tag{4}$$

This can be integrated over one cycle time to give the total mass (still seen from inside the micro cosmos) which is equation **(8)**:

$$m_{\mathrm{mc}} = \frac{1}{t_{\mathrm{crunch}}} \int_0^{t_{\mathrm{crunch}}} m_{\mathrm{mc}}(t)\, dt = \frac{\pi}{2} \cdot \frac{a_0}{a_{\mathrm{max}}} \cdot m_{\mathrm{Pl}} \tag{5}$$

Problems:

1. Use the definitions of ρ_r and V in the paper and derive equation **(6)**.
2. Explain how the mass can change with time using cosmological redshift/blueshift.
3. Follow the steps in section 4 and appendix A of the paper [1] and derive equation **(8)**.
4. Discuss whether the micro cosmos is a realistic model of a quantum.

3 The proton

The definition of the proton radius in the micro cosmos model, equation **(12)**, is:

$$r_{\mathrm{p}} \equiv 4 \cdot \frac{\hbar}{m_{\mathrm{p}} \cdot c} = 0.84123564119 \cdot 10^{-15}\ \mathrm{m} \tag{6}$$

Problems:

1. Insert the constants and verify the value of r_{p}.
2. What other explanations has been made for the large number coincidence?

4 Density of micro-cosmos

The density of the proton (nucleon), equation **(16)**, is:

$$\rho_{\rm nuc} \quad = \quad \frac{m_{\rm p}}{\frac{4\pi}{3}\cdot\left(4\cdot\frac{l_{\rm Pl}\cdot m_{\rm Pl}}{m_{\rm p}}\right)^3} = \frac{3}{4^4\cdot\pi}\cdot\left(\frac{m_{\rm p}}{m_{\rm Pl}}\right)^4\cdot\rho_{\rm Pl} \tag{7}$$

Problems:

1. Derive the expression for the nuclear density in equation **(16)**.
2. Insert the constants and verify that the value is consistent with equation **(15)**.
3. Describe the procedure of how the radial density is derived.
4. Discuss figure 1 and 2 in the paper and the prediction that the density in the center of a nucleon reaches the Planck density momentarily.
5. Discuss whether the classical method that is used (integration) would work if gravity, space and time were quantized.